Existence and Asymptotic Equivalence of Solutions to Nonlinear Volterra-Type Difference Equations

U. Ostaszewska, E. Schmeidel, B. Łupińska and M. Zdanowicz

We consider a Volterra-type difference equation, involving a general multiplicative coefficient $a(n)$ in the principal linear part, of the form

$$\Delta\big(r(n)(x(n+1)-a(n)x(n))\big)=b(n)+\sum_{k=1}^{n}K(n,k)f(x(k)), \tag{1}$$

where $n\in\mathbb{N}$, and the functions $r,a:\mathbb{N}\to\mathbb{R}\setminus\{0\}$, $b:\mathbb{N}\to\mathbb{R}$, the kernel $K:\mathbb{N}\times\mathbb{N}\to\mathbb{R}$, and the nonlinear function $f:\mathbb{R}\to\mathbb{R}$ satisfy appropriate assumptions.

We present sufficient conditions for the existence of solutions with prescribed asymptotic behavior. In particular, we show that solutions of the nonlinear equation admit asymptotic representations in terms of solutions of the corresponding linear equation, implying that the nonlinear Volterra term is asymptotically negligible. The obtained results extend several earlier contributions on the asymptotic theory of nonlinear difference equations to a broader class of Volterra-type equations with a general multiplicative coefficient in the principal linear part. The presented thesis are illustrated by representative examples.

J. Migda, M. Migda, Asymptotic behavior of solutions of discrete Volterra equations, Opuscula Mathematica, 36 (2), 265–278 (2016)

M. Migda, A. Dutkiewicz, Asymptotic behavior of solutions of second-order difference equations of Volterra type, Turkish Journal of Mathematics, 43 (5), 2203–2217 (2019)

First Author: Urszula, Ostaszewska
Affiliation: *Institute of Mathematics, University of Bialystok 15-245, Polska*
e-mail: `uostasze@math.uwb.edu.pl`

Second Author: Barbara, Łupińska
Affiliation: *Faculty of Computer Science, University of Bialystok 15-245, Polska*
e-mail: `b.lupinska@math.uwb.edu.pl`

Third Author: Ewa, Schmeidel
Affiliation: *Faculty of Computer Science, University of Bialystok 15-245, Polska*
e-mail: `e.schmeidel@uwb.edu.pl`

Fourth Author: Małgorzata, Zdanowicz
Affiliation: *Faculty of Computer Science, University of Bialystok 15-245, Polska*
e-mail: `malgorzata.zdanowicz@uwb.edu.pl`