Existence and location of the solutions to the singular eliptic problems in the exterior domain

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We present the results regarding the existence of positive solutions of the following singular problem with boundary condition

$$\begin{cases} \Delta u(x) + f(x, u(x)) - b(x)(u(x))^{-1}||\nabla u(x)||^2 + k(x, x \cdot \nabla u(x)) = 0, \quad x \in \Omega_R, \\ u(x) = S \text{ for } ||x|| = R, \\ \lim\limits_{||x|| \to \infty} u(x) = 0, \end{cases} \tag{1}$$

where $\Omega_R = \{x \in \mathbb{R}^n, ||x|| > R > 1\}$, $n \geq 3$, functions f, b, k are sufficiently smooth and $b(x) > 0$.

In the first part of the presentation, we discuss the history of similar problems ([1], [2]) and the existence of classical solutions to a problem with $b \equiv 0$. Our main tool is the Noussair-Swanson theorem concerning the sub-supersolution approach ([4]). Having the explicit formulas for both sub and supersolution established, we are able to provide the location of the solution u_S dependent on S. Then, we present the quantitative results for the class of problems similar to those described in [1].

In the second part, we show the existence of a positive solution to (1) using sub-supersolution method based on the work of S.Cui ([3]) for bounded sub-domains as well as the unbounded domain approximation method combined with some classical convergence procedure ([4], [5]). Location theorem is also presented for the singular case.

References

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