Equations with a generalized fractional Laplacian: different approaches

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Let $\Omega \subset \mathbb{R}^N$ be an open bounded set with Lipschitz boundary. We denote by

$$-\Delta : H_0^1(\Omega) \to H^{-1}(\Omega)$$

the weak Dirichlet Laplacian, defined by

$$\langle -\Delta u, v\rangle_{H^{-1}(\Omega)\times H_0^1(\Omega)} = \int_\Omega \nabla u \cdot \nabla v \, dx, \qquad u, v \in H_0^1(\Omega).$$

The operator $-\Delta$ has the following properties:

- it is self-adjoint and positive;
- its spectrum consists of a sequence of positive eigenvalues,

$$\sigma(-\Delta) = \{\lambda_n : n \in \mathbb{N}\};$$

- if $(e_n)_{n\in\mathbb{N}}$ is an orthonormal basis of eigenfunctions associated with $(\lambda_n)_{n\in\mathbb{N}}$, then

$$-\Delta u = \sum_{n=1}^{\infty} \lambda_n \langle u, e_n\rangle e_n, \qquad u \in D(-\Delta).$$

Accordingly, we define

$$g(-\Delta)u = \sum_{n=1}^{\infty} g(\lambda_n)\langle u, e_n\rangle e_n,$$

where $g : \sigma(-\Delta) \to \mathbb{R}$, with domain

$$D(g(-\Delta)) = \left\{ u \in L^2(\Omega) : \sum_{n=1}^{\infty} g(\lambda_n)^2 |\langle u, e_n\rangle|^2 < \infty \right\}.$$

We consider the equation

$$g(-\Delta)u = f(x, u),$$

where $f : \Omega \times \mathbb{R} \to \mathbb{R}$. We investigate the existence of solutions to this equation using different approaches.

References

[1] Igor Kossowski, Bogdan Przeradzki, Nonlinear equations with a generalized fractional Laplacian, RACSAM, 115 (2), art. 58 (2021).

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