Shannon Entropy as a Universal Metric for Uncertainty Quantification of Transport Phenomena and Phase Transitions in Computational Mechanics

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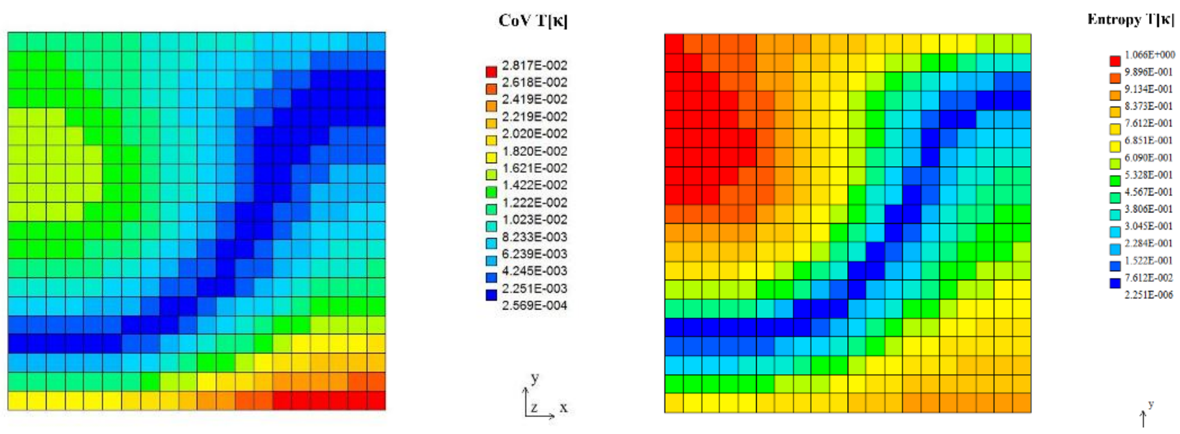
This work demonstrates the efficacy of Shannon entropy as a universal, information-theoretic metric for quantifying uncertainty propagation across diverse physical phenomena in computational mechanics [1]. Unlike traditional stochastic approaches that rely on decoupled, high-order statistical moments, probabilistic entropy compresses total system disorder into a single scalar field, offering critical insights into deterministic stability under input scattering.

We synthesize three distinct physical regimes from our recent research to document the versatility of this paradigm. First, in Stochastic Fluid Hydrodynamics, we utilize the Higher-Order Stochastic Finite Volume Method (SFVM) to show that extreme peaks of physical uncertainty align precisely with maximum coefficients of variation under Gaussian scattering of fluid viscosity and thermal conductivity (*Fig. 1*) [2]. Second, in Reaction-Diffusion Kinetics, utilizing the Stochastic Finite Difference Method (SFDM), we demonstrate that the temporal decay of Shannon entropy serves as a direct indicator of thermodynamic stabilization as processes approach macroscopic equilibrium [3]. Finally, in Microstructural Parameter Homogenization, Monte Carlo simulations of multi-phase nanofluids (Al_2O_3 and Cu) reveal that probabilistic entropy captures distinct non-linear fluctuations [4]. These highlight critical physical thresholds - such as nanoparticle clustering and percolation transitions - where minor volumetric fraction uncertainties cause disproportionate variations in effective properties.

Ultimately, this work establishes Shannon entropy as a powerful diagnostic tool for identifying spatial zones of increased thermal-hydraulic randomness, guiding the structural design of complex transport processes in uncertain physical environments.



(a) (b)

Figure 1. Coefficients of variation (a) and Shannon entropies (b) for the temperature field in the lid-driven cavity flow test with Gaussian thermal conductivity.

References:
[1] **Shannon, C.E.** A mathematical theory of computation. *Bell Systems Technical Journal* **1948**, *27*, 379–423 & 623–656.
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