A Multilevel Graph Neural Network Framework for Model Reduction in Contact Mechanics

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The numerical simulation of dynamic contact problems in viscoelastic solids is computationally demanding, particularly when high spatial resolution is required to accurately capture localized contact phenomena. This work presents a data-driven model reduction strategy that combines the Finite Element Method (FEM) with Graph Neural Networks (GNNs) to approximate fine-scale solutions at a significantly lower computational cost.

The proposed methodology employs a two-level discretization. The governing contact problem is solved on a coarse finite element mesh, while a graph neural network reconstructs the deformation of a corresponding fine mesh from the coarse-scale solution. To facilitate information exchange across discretization levels, we introduce a hierarchical graph representation with directed inter- and intra-mesh connections. The approach is developed for dynamic contact problems governed by a linear normal compliance law and Coulomb friction.

A normalization procedure based on rigid-body translation and rotation is incorporated to preserve the temporal coherence between coarse and fine mesh configurations throughout the simulation. The resulting framework is implemented in JAX and assessed by comparison with classical model reduction techniques, including Principal Component Analysis (PCA) and skinning-based interpolation methods.

The presented approach demonstrates that graph-based learning can serve as an effective approximation for high-dimensional finite element solutions, offering a promising direction for model order reduction in computational contact mechanics and other multiscale problems arising in continuum mechanics.

References

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